PROPOSAL FOR FRESHMAN SEMINAR: MATHEMATICS OF VOTING AND SOCIAL CHOICE

Instructor. Dr. Daniel Thompson, Associate Professor, Department of Mathematics.

Textbook. *A Mathematical Look at Politics*, Robinson and Ullman, CRC press, 2011

Content. The main theme of the seminar (around 70% of the course) will be a mathematical study of the principles of voting. A secondary theme (around 20% of the course) will discuss contemporary issues such as predicting elections, and what mathematicians can say about gerrymandering. This is a mathematics course, and a rigorous, idealized, study of political systems will be our focus rather than any detailed study of real world political science or statistics. The seminar will introduce some of the basic principles of mathematical reasoning. We will develop important skills used in pure mathematics, such as those introduced in the course "Math 3345 Fundamentals of Higher Mathematics". The final 10% of the course will pivot to these connections, through discussing axioms in Mathematics, and what properties follow from them.

(1) Principles of Voting. This seminar will be first and foremost a class about mathematical reasoning, motivated and illustrated by the world of politics. The student whose experience of mathematics so far has been the manipulation of equations will find a very different point of view taken here. We will create precise ways to characterize a decision making process, and investigate mathematically the characteristics of such a system. Perhaps the most famous result of this kind is Arrow's theorem, which in a precise and specific sense says that there is no fair way to have an election with more than two candidates. What does this mean for democracy? Can we interpret this result in the context of the 2020 Primary elections? It is important to note that we get to choose what we mean by 'a fair way to have an election', so perhaps Arrow's theorem is not as absolute as it seems on first glance. Nonetheless, this result illustrates the kind of mathematical issues that are in the background of our political life, and should be better understood. We will look at the following topics, focusing on an idealized study based on rigorous mathematical principles, rather than the nitty gritty of politics in the real world.

- What are the best systems for electing leaders and making democratic decisions? We could be talking about the US government, a university committee, a jury, or the board of directors of a company. Can mathematics help us decide the best voting system to choose in each of these contexts?

- What are the mathematical drawbacks of a given voting system? Do counter-intuitive outcomes occur under some of the most accepted voting systems? In an ideal world, could we devise a voting system that produces better outcomes?

- Apportionment: how does one draw the map of congressional districts in the fairest possible way? Even without political concerns such as gerrymandering, there are fundamental mathematical obstacles to a 'fair' system of apportionment. We will look at these obstacles, and understand the mathematical choices that need to be made.

- How does one resolve a conflict? We will look at some introductory game theory to help understand different strategies and outcomes for two or more parties that are in conflict.

(2) Mathematics in today's politics. A secondary goal of the seminar will be to look how mathematics used in contemporary politics. The 2020 election could be a pivotal year for the public reputation of data analysis in politics. It has been argued that a quantitative approach to predicting elections really came of age relatively recently, particularly in the 2008 and 2012 Presidential election cycles. The most high profile mathematical predictions of those elections were by Nate Silver. In 2008, he predicted the outcome of the Presidential election in 49 out of 50 states (and also correctly called all but one of the Senate races that year). In 2012, Nate Silver went one better and predicted all 50 states correctly!!!!!! His incredible predictions were a mixture of a very solid grasp on the mathematics, and the 'human angle' of factoring 'the reliability' of information into his models. However, the 2016 elections seriously damaged the reputation of this approach. Nate Silver predicted the 2016 election for Hilary Clinton, although he urged caution, and gave a more nuanced prediction than rivals including the NY Times, who had models predicting a Clinton win with 90% certainty. Will 2020 redeem the reputation of mathematical analysis of polling data? We will study this year's predictions as the semester unfolds, and we will be able to critique or give kudos to Nate Silver and his competitors when the results come in.

We will also talk about initiatives by Mathematicians to understand gerrymandering - e.g. by the Metric Geometry and Gerrymandering Group based at Tufts and MIT in the Boston area.

(3) Axioms in Mathematics. The activities of part (1) have much more to do with the study of pure mathematics than it appears on the surface. The starting point for pure Mathematics is a list of axioms defining the basic objects, and then one investigates what properties the objects defined have. We'll discuss axioms of the real numbers, and axioms in geometry. This will preview ideas the students will meet in advanced mathematics classes.

Learning goals. This seminar will be an introduction to the thought processes used in higher mathematics. We will give precise definitions of the political terms under consideration, and rigorously find out what one can deduce from the definitions. This approach has a lot in common with a course such as Math 3345 Foundations of Higher Mathematics, where one starts by defining the natural numbers, and begins the long process of building the rest of mathematics from the ground up. Students will end the seminar with a better understanding of the mathematical choices we make in our political structures, and will gain transferable skills which will be valuable if they choose to pursue a minor or major in mathematics.

Prerequisites and Target Audience. There are no formal prerequisites but the seminar will be particularly useful for students who are taking or considering a major or minor in mathematics. The seminar will also be useful for social scientists who are interested in improving their analytic background.

Organization. The seminar series will be taught as weekly one-hour workshops, where students will work in small groups to explore mathematical concepts based on politics. In addition, we will look at topical issues raised by this year's election cycle, and discuss how the mathematical properties of the voting systems we use effect real world politics. Additional reading, etc, will be required outside of class to prepare for the workshops. The expectation is that students will spend two hours each week outside of class working on the course. The textbook is suitable for students to independently dig deeper into the topics of the course. Here is a rough outline of the course.

Week 1: Introductory seminar.
Workshop written by instructor

Week 2: Voting for two candidates I: different systems.
Workshop based on Robinson and Ullman, §1.1-§1.3

Week 3: Voting for two candidates II: Criteria and May's theorem
Workshop based on Robinson and Ullman, §1.4, §1.5

Week 4: Voting for multiple candidates I: different systems.
Workshop based on Robinson and Ullman, §2.1, §2.2, §2.3

Week 5: Voting for multiple candidates II: Which criteria? Which systems are good?
Workshop based on Robinson and Ullman, §3 and §4

Week 6: Voting for multiple candidates III: Arrow's Impossibility Theorem
Workshop based on Robinson and Ullman, §5

Week 7: Apportionment I: Methods for Apportionment
Workshop based on Robinson and Ullman, §7 and §8

Week 8: Apportionment II: Criterion and Impossibility
Workshop based on Robinson and Ullman, §9

Week 9: Zero Sum Games: Intro to Game Theory
Workshop based on Robinson and Ullman, §13

Week 10: A mathematical look at the electoral college
Workshop based on Robinson and Ullman, §19-§20

Week 11: Five Thirty Eight and Beyond: Predicting Elections - Successes and Surprises
Workshop written by instructor

Week 12: Mathematics vs. Gerrymandering
Workshop written by instructor

Week 13: Axioms in Pure Mathematics: Real numbers; Geometries.
Workshop written by instructor

Assessment. The course will be graded S/U. Attendance is mandatory. Absences for legitimate reasons will be excused, although since we only meet once per week, should be kept to a minimum. Please save absences for illness or emergency. One absence will be excused routinely, and please notify me for additional absences (in advance if possible) or in case you are having difficulties. Homework questions will be informally set from the textbook to be prepared for discussion and feedback on solutions in the following class, requiring less than a page of write-up. Students will complete a dossier by the end of the course of between 8 and 10 pages. Intermediate work on the dossier will be collected at the midpoint of the semester (around week 7), and again two weeks before the end of the course (around week 11) for feedback. The final version should be turned in by the end of semester. The dossier can be built by compiling solutions from the weekly informal homeworks, or can include more detailed write-up of topics in the textbook or beyond (particularly in the second half of the course). Simply compiling the weekly suggested homeworks is one way to build a passing dossier. However, the format gives flexibility for students to alternatively complete their dossier by digging deeper into their choice of relevant topic if they so choose. Details of what should be included in each student's dossier will be approved through the semester on an individual basis in discussion with the Instructor. An S grade will be awarded based on attendance and turning in a final dossier completed to the standards of the course.

Academic Misconduct Statement. It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term academic misconduct includes all forms of academic misconduct wherever committed, illustrated by (but not limited to) cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the Committee. For additional information, please refer to the Code of Student Conduct, which can be found at `(http://studentaffairs.osu.edu/resource_csc.asp)`.

Disability Statement. The University strives to make all learning experiences as accessible as possible. If you anticipate or experience academic barriers based on your disability (including mental health, chronic or temporary medical conditions), please let me know immediately so that we can privately discuss options. To establish reasonable accommodations, I may request that you register with Student Life Disability Services. After registration, make arrangements with me as soon as possible to discuss your accommodations so that they may be implemented in a timely fashion. SLDS contact information: slds@osu.edu; 614-292-3307; `slds.osu.edu`; 098 Baker Hall, 113 W. 12th Avenue.

Brief biographical paragraph for the instructor. Dr. Daniel Thompson joined Ohio State in 2012. He is currently Associate Professor in the Department of Mathematics, working in an area called Ergodic Theory, which concerns the asymptotic behavior of systems that change over time. From 2009-2012, he was Chowla Research Assistant Professor at Penn State. Dr. Thompson completed his PhD in Mathematics at the University of Warwick in the UK in 2009. Dr. Thompson was awarded a five-year CAREER award in 2015, which is the top award given by the National Science Foundation to support the work of junior faculty members. Those selected "exemplify the role of teacher-scholars through outstanding research, excellent teaching and the integration of education and research". Dr. Thompson has taught undergraduate classes at Ohio State including Math 4547 Real Analysis I, Math 3345 Foundations of Mathematics, and Math 1181H Honors Calculus. Dr. Thompson was recently elected as an Alternate Member of the Ohio State University Faculty Senate, serving from 2020-2023.